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LUMINARY Memo # 62

TO: Distribution

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SUBJECT: A Set of Lunar Landing Guidance Equations which Compensate for Computation, Throttle, and Attitude Control Lags

Introduction

Attitude oscillations have been observed during the terminal part of automatic landings. A manual landing by the astronaut circumvents the attitude oscillations in P64 and P65. But it is desirable to build an automatic system from which the crew takes over because of preference not necessity. Allan Klumpp and Bill Widnall have concluded that the oscillations are due to the lags between the state vector times and the realization of the guidance commands by the two control systems, the engine throttle servo and the digital autopilot. Allan's engineering simulation appears to confirm this. This memo gives a set (which ought to turn out to be somehow equivalent to Allan's) of lunar landing guidance equations which provide lead compensation for the lags.

Definition

Here are some definitions necessary to understand the rest of this memo.

t_o = state vector time, the time at which the PIPA's are read.

$\underline{r}_o$, $\underline{v}_o$ = the position and velocity at t_o, i.e. $\underline{r}(t_o)$, $\underline{v}(t_o)$.

t^* = $t_o + \tau$, a time τ seconds later than state vector time, but never greater than T. τ is the amount of time the guidance equations are projected into the future from state vector time.

T = the time at which the desired position, velocity, and acceleration vectors are to be achieved.

$\underline{r}_D, \underline{v}_D, \underline{a}_{TD}$ = the position, velocity and thrust acceleration vectors, desired at T.

T_{go} = the "time-to-go". The time between t_o and T. Thus, $T_{go} = T - t_o$.

T^*_{go} = the "diminished" time-to-go, which tries to take into account the lags between t_o and the realization of guidance commands.

$$T^*_{go} = T - t^* = T_{go} - \tau$$

$\underline{g}_o, \underline{g}(T)$ = the gravitation acceleration at state vector time (output of AVERAGEG) and at T, respectively.

$$\underline{g}(T) = -\mu \underline{r}_D / \left| \underline{r}_D \right|^3$$

$\underline{a}_{TC}(t^*)$ = the guidance equations commanded thrust acceleration τ seconds after state vector time.

$\underline{a}, \underline{b}, \underline{c}$ = three guidance parameters which are functions of T_{go}, the state at t_o, the desired state at T, and the desired thrust acceleration vector at T. These are such that the exact solution to the guidance problem is given by

$$\underline{a}_{TC}(t) = \underline{a} + (T-t)\,\underline{b} + (T-t)^2\,\underline{c} - \underline{g}(t)$$

This equation is valid for the entire future and the guidance parameters change only because of navigation errors, terrain irregularities, control errors, re-targetting. Realistically, the guidance parameters change very slowly.

The Equations

The equations are given here for those who are interested in the computational aspects without wading through the derivation. A second memo will give my derivation of the equation. The equations are given in steps which the computer program might take.

1. Determine $\underline{r}_o$, $\underline{v}_o$ and t_o. That is, read the computer clocks, the PIPA's, and perform the AVERAGEG equations.

2. Compute $\underline{a}$, the first guidance parameter

$$\underline{a} = \underline{a}_{TD} + \underline{g}(T)$$

(Note: This does not have to be computed every two seconds; but only when the targetting changes.)

3. Compute

$$T_{go} = T - t_o$$

4. Compute the following functions of state at T and t_o

$$\underline{x} = \underline{v}_D - \underline{v}_o - T_{go}\,\underline{a}$$

$$\underline{y} = \underline{r}_D - \underline{r}_o - T_{go}\underline{v}_o - (T_{go}^2/2)\underline{a}$$

5. Compute the 2nd and 3rd guidance parameters.

$$\underline{b} = (18/T_{go}^{\;2})\,\underline{x} - (24/T_{go}^{\;3})\,\underline{y}$$

$$\underline{c} = -(24/T_{go}^{\;3})\,\underline{x} + (36/T_{go}^{\;4})\,\underline{y}$$

6. Compute the command thrust acceleration vector corresponding to t^*.

$$T_{go}^* = T_{go} - \tau \quad (\text{if } T_{go}^* > 0;\ \text{otherwise } T_{go}^* = 0)$$

$$\underline{a}_{TC}(t^*) = \underline{a} + T_{go}^*\,\underline{b} + (T_{go}^*)^2\,\underline{c} - \underline{g}_o$$

(Note: $\underline{g}_o$ above should really be $\underline{g}(t^*)$; but the $\underline{g}(t)$ vector changes negligibly in 3 seconds, the approximate proposed value for $\tau = t^* - t_o$.)

7. Compute the desired thrust acceleration magnitude and direction.

$$\underline{\text{direction}} = \text{unit}\,[\underline{a}_{TC}\,(t^*)]$$

$$\text{magnitude} = \text{abval}\,[\,\underline{a}_{TC}(t^*)]$$

and output these commands.

A few words are in order about the adavntages of these equations.

When T_{go} becomes small the navigation noise on $\underline{r}_o$ and $\underline{v}_o$ will begin to make $\underline{b}$ and $\underline{c}$ go "wild" (because of the rapidly increasing gains on $\underline{x}$ and $\underline{y}$ in step 5). But $\underline{a}$, $\underline{b}$, and $\underline{c}$ should be constants (if navigation and control are perfect) and step 6 can be performed with $\underline{b}$ and $\underline{c}$ computed five or ten seconds or even longer in the past. Thus, when T_{go} becomes less than, say, ten seconds, steps 4 and 5 can be skipped and step 6 used to project the desired thrust acceleration vector into the future.

Another consideration which this form of the equations makes possible is the relaxation of final position constraint at a certain T_{go}, but the continued maintenance of the velocity constraint at a smaller T_{go}.

In steps 6 and 7, I showed the same t^* for projecting both the direction (input to DAP) and magnitude (input to engine throttle servo); but for very little additional coding a different T_{go}^* and $\underline{a}_{TC}(t^*)$ can be generated for the commands to the DAP and throttle. This has virtue if the time constants (to which τ is related) are very different.

Note, finally, that we could generate an analytically perfect $\underline{w}_D$ and d (magnitude)/dt. The $\underline{w}_D$ could drive the DAP.

The above equations make no approximations except for the slight one noted in step 6. But I may have blundered algebraically in the derivation. The derivation will follow in the next memo and confirm or correct the equations.